

**INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH
TECHNOLOGY****RELIABILITY AND PROFIT EVALUATION OF COMPRESSOR SYSTEM
DESCRIBING FAILURES AND DEAL WITH FAILED UNIT ON PRIORITY****Dr. Upasana Sharma *, Jaswinder Kaur*** Associate Professor , Department of Statistics, Punjabi University Patiala, India
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ABSTRACT

The paper presents reliability and profit evaluation of compressor standby system comprising of two units. Initially one unit is operative and the other is in standby state. On failure of any one of the compressor unit, the standby unit becomes operative to keep the system in operating state. If both the compressor units get failed then system goes to down state. Here compressor unit can fail due to three types of failure which are serviceable, repairable and replaceable type. Out of two compressor units, the unit which fail earlier will get facility of service, repair or replacement. System is analyzed and expressions for various reliability measures such as MTSF, Availability, Busy periods for various types of failures and profit etc have been computed numerically by using semi-Markov process and regenerative point technique. Graphs for profit are plotted for making the present study more useful.

KEYWORDS: Compressor unit, Standby system, Regenerative point technique, semi-Markov process.**INTRODUCTION**

In milk plants for functioning of refrigeration system compressor standby system act as a vital organ. Upon the failure of this system, it has been observed that working of refrigeration system get effected seriously. In field of reliability standby systems have been analyzed by large number of researchers such as [1]-[7]. Compressor standby system is still untouched in the field of reliability. The present paper is our genuine effort to study such system and to fill this research gap. Paper present reliability and profit evaluation of compressor standby system comprising of two units. Initially one unit is operative and the other is standby state. On failure of any one of the compressor unit, the standby unit becomes operative to keep the system in operating state. If both the compressor units get failed then system goes to down state. Here compressor unit can fail due to three types of failure which are serviceable, repairable and replaceable type. Out of two compressor units, the unit which fail earlier will get facility of service, repair or replacement. Upon failure of second compressor unit if the first compressor unit is still in service, repair or replacement then this unit will be kept in waiting state for service, repair or replacement. System is analyzed and expressions for various reliability measures such as MTSF, Availability, Busy periods for service, Busy period for repair, Busy period for replacement and profit etc have been computed numerically by using semi-Markov process and regenerative point technique. Graphs for profit are plotted for making the present study more useful. For profit purpose the unit real failure as well as repair time data from a milk plant have been collected.

NOTATIONS

O_I	First Compressor is in Operative State
S_{II}	Second Compressor is in Standby state
F_{sI}, F_{sII}	Failure category of Serviceable type for First, and Second compressor
F_{rI}, F_{rII}	Failure category of Repairable type for First and Second compressor
F_{repI}, F_{repII}	Failure category of Replaceable type for First and Second compressor
$F_{wII}, F_{wsII}, F_{wrepII}$	Second compressor is waiting for Repair, Service and Replacement respectively
$F_{uI}, F_{urI}, F_{urepI}$	First compressor is under Service, Repair and Replacement respectively

$\lambda_{i1}, \lambda_{i2}, \lambda_{i3}$	Failure rate when failure is of Serviceable , Repairable and Replaceable for First and Second compressor respectively (i= I, II and i symbol used for compressor unit)
$\alpha_{i1}, \alpha_{i2}, \alpha_{i3}$	Repair rates when failure is of Serviceable , Repairable and Replaceable type for First and Second compressor respectively
$G_{i1}(t), g_{i1}(t)$	c.d.f and p.d.f of time for Service when failure is of Serviceable type for First and Second compressor respectively
$G_{i2}(t), g_{i2}(t)$	c.d.f and p.d.f of time for Repair when failure is of Repairable type for First and Second compressor respectively
$G_{i3}(t), g_{i3}(t)$	c.d.f and p.d.f of time for Replacement when failure is of Replaceable type for First and Second compressor respectively
Q_{ij}, q_{ij}	c.d.f and p.d.f of first passage time from a regenerative state i to j or to a failed state j in (0, t].
$\Phi_i(t)$	c.d.f of the first passage time from regenerative state i to a failed state
P_{ij}, P_{ij}^k	probability of transition from regenerative state i to regenerative state j without visiting any other state in (0,t],visiting state k once in (0,t]
q_{ij}^k	p.d.f of first passage from regenerative state i to regenerative state j or to failed state j visting k once in (0,t]
(s)	Stieltjes convolution
©	Laplace convolution

MODEL DESCRIPTION AND ASSUMPTIONS

- 1) State 0 is the initial operative state and transition from this state to states 1,2, and 3 depends on type of failure.
- 2) All failure times are exponentially distributed.
- 3) After each service/ repair/replacement compressor unit works as good as new.
- 4) Priority given to failed unit for service , repair and replacement.

Table 1: Possible states with status

State No.	Status	State No.	Status
0	O_I, S_{II}	8	F_{urI}, F_{wrII}
1	F_{sI}, O_{II}	9	F_{urI}, F_{wrepII}
2	F_{rI}, O_{II}	10	F_{urepI}, F_{wsII}
3	F_{repI}, O_{II}	11	F_{urepI}, F_{wrII}
4	F_{usI}, F_{wsII}	12	F_{urepI}, F_{wrepII}
5	F_{usI}, F_{wrII}	13	O_I, F_{sII}
6	F_{usI}, F_{wrepII}	14	O_I, F_{rII}
7	F_{urI}, F_{wsII}	15	O_I, F_{repII}

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The possible state transition are shown in Table. 1.The epochs of entry into states 0,1,2,3,13,14 and 15 are regenerative states. States 4,5,6,7,8,9,10,11 and 12 are down states. The non zero elements p_{ij} are given below:

$$p_{01} = \frac{\lambda_{11}}{\lambda^*}, p_{02} = \frac{\lambda_{12}}{\lambda^*}, p_{03} = \frac{\lambda_{13}}{\lambda^*} \text{ where } \lambda^* = \lambda_{11} + \lambda_{12} + \lambda_{13}, p_{10} = g_{11}^*(\lambda), p_{20} = g_{12}^*(\lambda), p_{30} = g_{13}^*(\lambda) \text{ where}$$

$$p_{27}, p_{2,13}^7 = \frac{\lambda_{21}}{\lambda} (1 - g_{12}^*(\lambda)); p_{28}, p_{2,14}^8 = \frac{\lambda_{22}}{\lambda} (1 - g_{12}^*(\lambda)); p_{29}, p_{2,15}^9 = \frac{\lambda_{23}}{\lambda} (1 - g_{12}^*(\lambda)); p_{14}, p_{1,13}^4 = \frac{\lambda_{21}}{\lambda} (1 - g_{11}^*(\lambda))$$

$$p_{15}, p_{1,14}^5 = \frac{\lambda_{22}}{\lambda} (1 - g_{11}^*(\lambda)); p_{16}, p_{1,15}^6 = \frac{\lambda_{23}}{\lambda} (1 - g_{11}^*(\lambda)); p_{3,10}, p_{3,13}^{10} = \frac{\lambda_{21}}{\lambda} (1 - g_{13}^*(\lambda)); p_{3,11}, p_{3,14}^{11} = \frac{\lambda_{22}}{\lambda} (1 - g_{13}^*(\lambda))$$

$$p_{3,12}, p_{3,15}^{12} = \frac{\lambda_{23}}{\lambda} (1 - g_{13}^*(\lambda)), p_{10} + p_{14} + p_{15} + p_{16} = 1, p_{10} + p_{1,13}^4 + p_{1,14}^5 + p_{1,15}^6 = 1, p_{20} + p_{27} + p_{28} + p_{29} = 1$$

$$p_{20} + p_{2,13}^7 + p_{2,14}^8 + p_{2,15}^9 = 1, p_{30} + p_{3,10} + p_{3,11} + p_{3,12} = 1, p_{30} + p_{3,13}^{10} + p_{3,14}^{11} + p_{3,15}^{12} = 1, \lambda = \lambda_{21} + \lambda_{22} + \lambda_{23}$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$\mu_0 = \frac{1}{\lambda_{11} + \lambda_{12} + \lambda_{13}}, \mu_1 = \frac{1}{\lambda_{21} + \lambda_{22} + \lambda_{23}}, \mu_2 = \frac{1}{\lambda_{21} + \lambda_{22} + \lambda_{23}}, \mu_3 = \frac{1}{\lambda_{21} + \lambda_{22} + \lambda_{23}}$$

$$\mu_4 = \int_0^{\infty} \bar{G}_{21}(t) dt, \mu_5 = \int_0^{\infty} \bar{G}_{22}(t) dt, \mu_6 = \int_0^{\infty} \bar{G}_{23}(t) dt, \mu_7 = \int_0^{\infty} \bar{G}_{21}(t) dt, \mu_8 = \int_0^{\infty} \bar{G}_{22}(t) dt, \mu_9 = \int_0^{\infty} \bar{G}_{23}(t) dt$$

$$\mu_{10} = \int_0^{\infty} \bar{G}_{21}(t) dt, \mu_{11} = \int_0^{\infty} \bar{G}_{22}(t) dt, \mu_{12} = \int_0^{\infty} \bar{G}_{23}(t) dt, \mu_{13} = \int_0^{\infty} \bar{G}_{21}(t) dt, \mu_{14} = \int_0^{\infty} \bar{G}_{22}(t) dt, \mu_{15} = \int_0^{\infty} \bar{G}_{23}(t) dt$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

$$m_{ij} = \int_0^{\infty} t dQ_{ij}(t) = -q_{ij}^*(0)$$

$$m_{01} + m_{02} + m_{03} = \frac{1}{(\lambda^*)} = \mu_0, m_{10} + m_{14} + m_{15} + m_{16} = \mu_1(1 - g_{11}^*(\lambda)), m_{20} + m_{27} + m_{28} + m_{29} = \mu_2(1 - g_{12}^*(\lambda))$$

$$m_{30} + m_{3,10} + m_{3,11} + m_{3,12} = \mu_3(1 - g_{13}^*(\lambda))$$

MEAN TIME TO SYSTEM FAILURE

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system as absorbing states. Now mean time to system failure (MTSF) when unit started at the beginning of state 0 is

$$MTSF = T_0 = \lim_{s \rightarrow 0} \frac{1 - \phi_0^*(s)}{s} = \frac{N}{D}$$

$$\text{Where } N = -\mu_0 + m_{01}p_{10} + m_{10}p_{01} + p_{20}m_{02} + p_{02}m_{20} + p_{30}m_{03} + p_{03}m_{30} - \mu_1 p_{01}(1 - g_{11}^*(\lambda)) - \mu_2 p_{02}(1 - g_{12}^*(\lambda)) - \mu_3 p_{03}(1 - g_{13}^*(\lambda))$$

$$D = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}$$

AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at t=0. In steady state availability of the system is given by

$$A_0 = \lim_{s \rightarrow 0} (sA_0^*(s)) = N_1 / D_1$$

where

$$N_1 = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} + \mu_{13} p_{01} p_{1,13}^4 + \mu_{14} p_{01} p_{1,14}^5 + \mu_{15} p_{01} p_{1,15}^6 + \mu_{13} p_{02} p_{2,13}^7 + \mu_{14} p_{02} p_{2,14}^8$$

$$+ \mu_{15} p_{02} p_{2,15}^9 + \mu_{13} p_{03} p_{3,13}^{10} + \mu_{14} p_{03} p_{3,14}^{11} + \mu_{15} p_{03} p_{3,15}^{12}$$

$$D_1 = \mu_0 + p_{01} \mu_1 (1 - g_{11}^*(\lambda)) + p_{02} (\mu_2 (1 - g_{12}^*(\lambda))) + (p_{03} \mu_3 (1 - g_{13}^*(\lambda))) + m_{13,0} (p_{01} p_{1,13}^4 + p_{02} p_{2,13}^7$$

$$+ p_{03} p_{3,13}^{10}) + m_{14,0} (p_{01} p_{1,14}^5 + p_{02} p_{2,14}^8 + p_{03} p_{3,14}^{11}) + m_{15,0} (p_{01} p_{1,15}^6 + p_{02} p_{2,15}^9 + p_{03} p_{3,15}^{12})$$

Proceeding in the similar fashion as above following measures in steady state have also been obtained

BUSY PERIOD ANALYSIS FOR SERVICE TIME ONLY	$B_0 = N_2 / D_1$
BUSY PERIOD ANALYSIS FOR REPAIR TIME ONLY	$B_1 = N_3 / D_1$
BUSY PERIOD ANALYSIS FOR REPLACEMENT TIME ONLY	$B_2 = N_4 / D_1$
EXPECTED NUMBER OF SERVICE	$S_E = N_5 / D_1$
EXPECTED NUMBER OF REPAIRS	$R_E = N_6 / D_1$
EXPECTED NUMBER OF REPLACEMENTS	$R = N_7 / D_1$
EXPECTED NUMBER OF VISITS BY REPAIRMAN	$V_0 = N_8 / D_1$

Where

$$N_2 = p_{01}k_1 + p_{01}p_{1,13}^4k_{13} + p_{02}p_{2,13}^7k_{13} + p_{03}p_{3,13}^{10}k_{13}$$

$$N_3 = p_{02}k_2 + p_{01}p_{1,14}^5k_{14} + p_{02}p_{2,14}^8k_{14} + p_{03}p_{3,14}^{11}k_{14}$$

$$N_4 = p_{03}k_3 + p_{01}p_{1,15}^6k_{15} + p_{02}p_{2,15}^9k_{15} + p_{03}p_{3,15}^{12}k_{15}$$

$$N_5 = p_{01} + p_{01}(p_{1,13}^4 + p_{1,14}^5 + p_{1,15}^6) + p_{01}p_{1,13}^4p_{13,0} + p_{02}p_{2,13}^7p_{13,0} + p_{03}p_{3,13}^{10}p_{13,0}$$

$$N_6 = p_{02} + p_{02}(p_{2,13}^7 + p_{2,14}^8 + p_{2,15}^9) + p_{02}p_{1,14}^5p_{14,0} + p_{02}p_{2,14}^8p_{14,0} + p_{03}p_{3,14}^{11}p_{14,0}$$

$$N_7 = p_{03} + p_{03}(p_{3,13}^{10} + p_{3,14}^{11} + p_{3,15}^{12}) + p_{01}p_{1,15}^6p_{15,0} + p_{02}p_{2,15}^9p_{15,0} + p_{03}p_{3,15}^{12}p_{15,0}$$

$$N_8 = p_{01} + p_{02} + p_{03}$$

$$k_{13} = \int_0^{\infty} \bar{G}_{21}(t) dt, k_{14} = \int_0^{\infty} \bar{G}_{22}(t) dt, k_{15} = \int_0^{\infty} \bar{G}_{23}(t) dt$$

For graphical representation, let us suppose that

$$g_{11}(t) = \alpha_{11}e^{-\alpha_{11}t}, g_{12}(t) = \alpha_{12}e^{-\alpha_{12}t}, g_{13}(t) = \alpha_{13}e^{-\alpha_{13}t}$$

using the above particular case, the following values are estimated as

$$\alpha_{11} = 0.006896, \alpha_{12} = 0.000586, \alpha_{13} = 0.04166, \alpha_{21} = 0.0000983, \alpha_{22} = 0.0001347, \alpha_{23} = 0.00015873, \lambda_{11}, \lambda_{12}, \lambda_{13} = 0.00003868$$

$$\lambda_{21}, \lambda_{22}, \lambda_{23} = 0.0007352, C_0 = 100, C_1 = 3000, C_2 = 500, C_3 = 550, C_4 = 800, C_5 = 27700, C_6 = 7600, C_7 = 7975$$

CONCLUSION

Mean time to unit/compressor MTSF = 13937.512 hrs.

Availability of the unit/compressor (A_0) = 1.0000000

Busy period analysis for service time only (B_0) = 0.158270

Busy period analysis for repair time only (B_1) = 0.153616

Busy period analysis for replacement time only (B_2) = 0.096448

Expected number of services (S_E) = 0.0000624833

Expected number of repairs (R_E) = 0.0000645

Expected number of replacements (R) = 0.00006321

PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2B_1 - C_3B_2 - C_4V_0 - C_5S_E - C_6R_E - C_7R$$

Where

C_0 = Revenue per unit up time

C_1 = Cost per unit time for which repairman is busy for service

C_2 = Cost per unit time for which repairman is busy for repair

C_3 = Cost per unit time for which repairman is busy for replacement

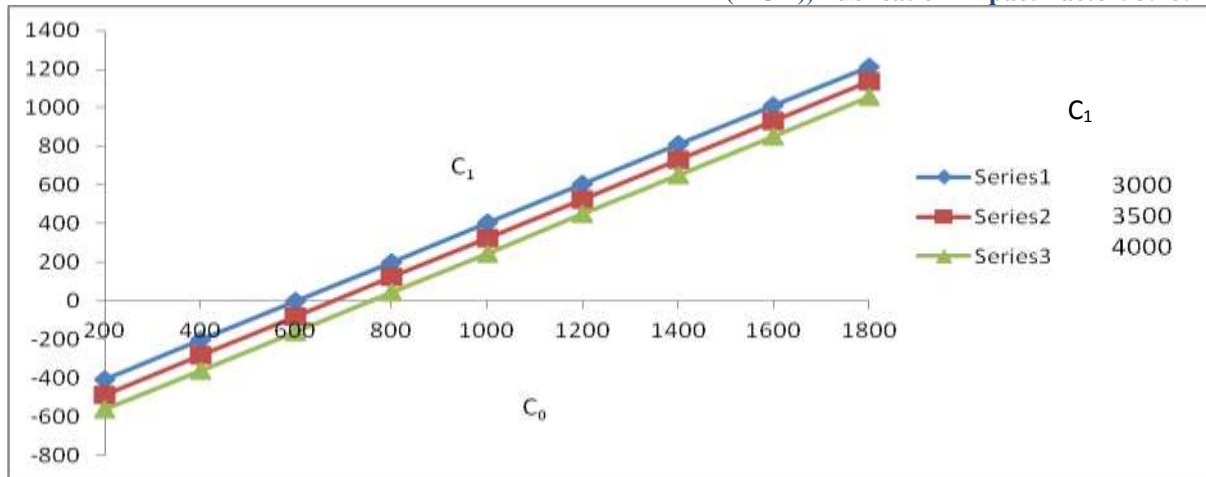
C_4 = Cost per visit of Repairman

C_5 = Cost per service

C_6 = Cost per Repair

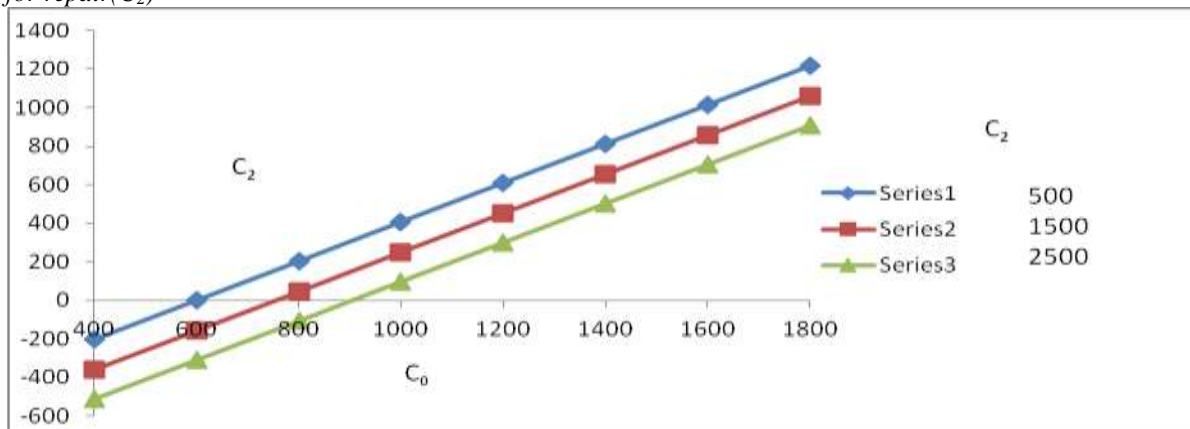
C_7 = Cost per Replacement

Graph between Profit vs Revenue per unit up time (C_0) for different values of cost per unit for which repairman is busy for service (C_1)



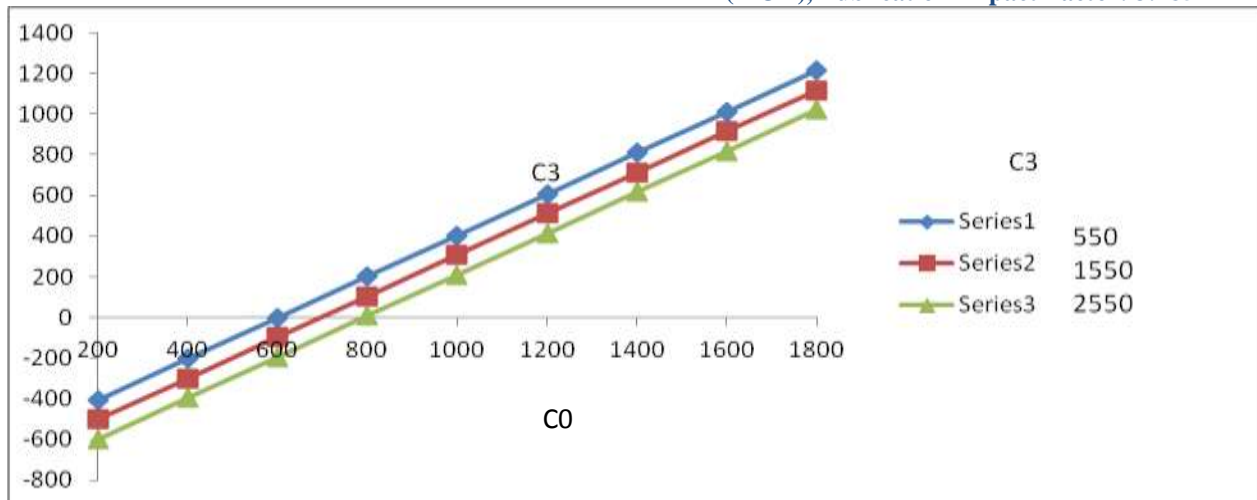
It is concluded from graph that profit increases with increase in values of revenue per unit up time (C_0). It can also be noticed that if $C_1=3000$, then $P > \text{or} = \text{or} < 0$ according as $C_0 > \text{or} = \text{or} < 607.5$. So for $C_1=3000$, revenue per unit up time should be fixed greater than 607.5. Similarly for $C_1=3500$ and 4000, the revenue per unit up time should be greater than 686.6 and 765.8 respectively.

Graph between Profit vs Revenue per unit time (C_0) for different values of cost per unit for which repairman is busy for repair (C_2)



It is concluded from graph that profit increases with increase in values of revenue per unit up time (C_0). It can also be noticed that if $C_2=500$, then $P > \text{or} = \text{or} < 0$ according as $C_0 > \text{or} = \text{or} < 607.5$. So for $C_2=500$, revenue per unit up time should be fixed greater than 607.5. Similarly for $C_2=1500$ and 2500, the revenue per unit up time should be greater than 761.1 and 914.7 respectively.

Graph between Profit vs Revenue per unit time (C_0) for different values of cost per unit for which repairman is busy for replacement (C_3)



It is concluded from graph that profit increases with increase in values of revenue per unit up time (C_0). It can also be noticed that if $C_3=550$, then $P \geq 0$ according as $C_0 \geq 607.5$. So for $C_3=550$, revenue per unit up time should be fixed greater than 607.5. Similarly for $C_3=1550$ and 2550, the revenue per unit up time should be greater than 703.9 and 800.4 respectively.

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